A New Approach of Solving Regional Water Supply Management Problem by Using Extended Nonlinear Transportation Model

Honghai Qi\textsuperscript{1}, Mustafa S. Altinakar\textsuperscript{2}, Bahram Alidaee\textsuperscript{3}

ABSTRACT: An extended nonlinear model based on basic transportation model is proposed here to solve local water supply management problem. This model considers various physical, financial and environmental factors faced in a typical water supply system. A hypothetical case study followed by the solution technique demonstrates the advantages that are gained by using Nelder-Mead Algorithm while the commercial software CPLEX\textsuperscript{®} fails. The near optimal result shows this algorithm is effective and promising in solving such model.

INTRODUCTION

Meeting the increasing demands for a public portable water supply is a complex multi-jurisdictional problem. It involves not only developing new supply facilities, but also operating them in a beneficial way so as to minimize the total cost. Regional water management system usually includes great flexibility in the choice of alternative sources, either surface water or groundwater, and conservation of environmental and economic resources for the region as well. For many years, engineers and water conservation and supply agencies are faced with this realistic problem.

Mathematical optimization techniques are an important tool in solving problems faced in regional water resources management. In the region with different alternatives, these techniques enhance the quality of the decision-making process. In recent decades, water supply management studies have extensively focused on the application of these techniques such as LP, DP or NLP to determine operation policies. Reviews of the previous work can be found in works by Sun, et al (1995), Randall et al (1997).

The present paper explores the same type of problem from a new prospective. First a water supply management model is built starting from a basic transportation model. The model is then improved by including various financial, physical and environmental factors. The resulting complicated nonlinear model is applied to a hypothetical case, and the Nelder-Mead derivative-free search algorithm is implemented to solve the model while CPLEX software doesn’t work. The near optimal results show that this algorithm is effective and promising in solving the proposed model.

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TRANSPORTATION MODEL

The study of transportation problem to serve clients at minimum cost has been one of the most studied themes in Operation Research. This type of problem involves the transport of goods from \( m \) “origins” to \( n \) “destinations” at minimal cost. According to Winston (2004), the classical transportation model can be described as follows. There is a set of supply points from which a good is shipped; each of them has at most \( CaP_i \) units’ production capacity. And there is also a set of demand points to which the good is shipped; each of them must receive \( D_j \) units of the shipped good. Each unit produced at supply point \( i \) and shipped to demand point \( j \) incurs a variable cost of \( c_{ij} \). The service goal is to determine an assignment of clients to these facilities without violating the capacity constraints so as to minimize the total transportation cost.

Since the physical configuration of a regional water supply system can be characterized as a network, transportation model can be built to decide how to “transport” water to each city at minimal cost. Let \( x_{ij} \) to denote the amounts of water (in million gallons) transported from plant \( i \) to city \( j \). The vector \( c_{ij} \) ($) represents the cost of transporting 1 million gallons of water from each plant \( i \) to each city. Also according to different pipe conditions (size, age and etc.), maintenance costs and the pumping capacities, \( RU_{ij} \) is used to denote the corresponding upper limit of transportation quantities from plant \( i \) to city \( j \) (Greenberg et al, 1977).

The transportation model can then be formulated as:

Objective Function (Minimize the total cost):

\[
\text{Min} \sum_{i} \sum_{j} c_{ij} x_{ij}
\]

Subject to:

\[
\sum_{j} x_{ij} \leq CaP_i, \text{ for plant } i \quad \text{(Supply constraints)}
\]

\[
\sum_{i} x_{ij} = D_j, \text{ for city } j \quad \text{(Demand constraints)}
\]

\[
x_{ij} \leq RU_{ij}, \text{ for all } i \text{ and } j \quad \text{(Capacity constraints)}
\]

\[
x_{ij} \geq 0
\]

EXTENDED MODEL FORMULATION

Fixed Budget Problem

If the city only has a fixed budget \( M \) for doing this project, it may not be possible to find a feasible solution which meets all demands. When total supply is less than total demand,
it is sometimes desirable to allow the possibility of leaving some demand unmet. In such a situation, a penalty cost \( P_j \) is often associated with unmet demand (Winston, 2004). In this case a supplementary objective function should be designed in order to maximize the supply that can be satisfied (or to minimize the shortage):

**Objective Function (Minimize the penalty):**

\[
\text{Min } \sum_j P_j(D_j - \sum_i x_{ij})
\]

The second constraint of the original model should also be rewritten in the following form:

\[
\sum_i x_{ij} \leq D_j, \text{ for city } j
\]

**Source and Bending Problem**

For the source of the water treatment plant, we may have several choices, such as the nearby reservoirs or the underground wells. Let us assume that two sources are available in the region. In the present case, production cost for one million gallons of water is not a fixed quantity. The following constraint equations need to be designed:

\[
v_i = \frac{E_a a_i}{100} + \frac{E_b b_i}{100}, \text{ for plant } i
\]

and \( a_i + b_i = 100 \), for plant \( i \)

where \( v_i \) is the variable production cost for each plant. \( a_i \) denotes the percent amount of water drawn from the reservoir and \( b_i \) denotes the percent amount of water pumping from the wells for each plant \( i \) (per 1 million gallon); \( E_a \) and \( E_b \) are the corresponding production cost for using water from reservoir and ground wells (per 1 million gallon).

Traditional groundwater sources are expected to remain relatively inexpensive while the surface water supplies are projected to be slightly more expensive, largely due to higher costs of treatment. Two additional constraints should be considered, due to the fact that the total amount of water drawn from these two sources should be limited to their maximum supply capacities:

\[
\sum_i \left( \frac{1}{100} a_i \right) \left( \sum_j x_{ij} \right) \leq \text{Max}(R)
\]

\[
\sum_i \left( \frac{1}{100} b_i \right) \left( \sum_j x_{ij} \right) \leq \text{Max}(P)
\]
where \( Max(R) \) denotes the maximum reservoir release and \( Max(P) \) denotes the maximum pumping capacity from the underground wells.

Hardness of the water from the groundwater and surface water resources can be quite different. Generally speaking, the ground water has higher hardness concentration than surface water. But the system should keep water hardness from exceeding a standard concentration. One way of ensuring that mixed water leaving the blending facility has acceptable hardness content is to set a maximum hardness content as a constraint. (Randall et al., 1997)

\[
H_a \left( \frac{a_i}{100} \right) + H_b \left( \frac{b_j}{100} \right) \leq H_{stan} \left( \sum_i x_{ij} \right) \quad \text{for plant } i
\]

Here \( H_a \) and \( H_b \) denote the hardness concentration of the water from the reservoir and from the groundwater and the \( H_{stan} \) denotes the standard hardness concentration of the water to be meet (in ppm). Then the above equation can be simplified as:

\[
H_a \left( \frac{a_i}{100} \right) + H_b \left( \frac{b_j}{100} \right) \leq H_{stan} \quad \text{for plant } i
\]

**Integrated Model Formulation**

We now summarize the above-described formulation procedure for the proposed planning and management model. As it is seen below, we have there main objectives: minimize the total transportation cost + variable production cost + shortage penalty cost subject to various financial, physical and environmental constraints:

**Objective Function:**

\[
\text{Min} \sum_i \sum_j c_{ij} x_{ij} + \sum_i \left( \frac{E_a a_i}{100} + \frac{E_b b_j}{100} \right) \left( \sum_j x_{ij} \right) + \sum_j P_j \left( D_j - \sum_i x_{ij} \right)
\]

**Subject to:**

\[
\sum_j x_{ij} \leq Ca P_i, \quad \text{for each plant } i
\]

\[
\sum_i x_{ij} \leq D_j, \quad \text{for each city } j
\]

\[
x_{ij} \leq RU_{ij}, \quad \text{for all } i \text{ and } j
\]

\[
\sum_i \left( \frac{1}{100} a_i \right) \left( \sum_j x_{ij} \right) \leq Max(R)
\]

\[
\sum_i \left( \frac{1}{100} b_i \right) \left( \sum_j x_{ij} \right) \leq Max(P)
\]

\[
a_i + b_i = 100, \quad \text{for each plant } i
\]
HYPOTHETICAL CASE

Let there be 5 cities with growing water demand within the planning period (for one year). Table 1 shows the amount of water supply that must be satisfied.

Table 1 Water demands for city 1 to 5

<table>
<thead>
<tr>
<th>City (j)</th>
<th>Water Demand (D_j) (million gallons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>180</td>
</tr>
<tr>
<td>2</td>
<td>120</td>
</tr>
<tr>
<td>3</td>
<td>140</td>
</tr>
<tr>
<td>4</td>
<td>120</td>
</tr>
<tr>
<td>5</td>
<td>260</td>
</tr>
</tbody>
</table>

Since the current treatment facilities cannot provide those amounts of water, the cities jointly decided to build some new water treatment plants. Based on the preliminary investigations, 3 plant locations were chosen. Their production capacities are given as:

Table 2 Production capacities for plant 1 to 3

<table>
<thead>
<tr>
<th>Plant (i)</th>
<th>Max Capacity (CaP_i) (million gallons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>420</td>
</tr>
<tr>
<td>2</td>
<td>380</td>
</tr>
<tr>
<td>3</td>
<td>260</td>
</tr>
</tbody>
</table>

According to the distance between the cities and the water treatment plants and also the needed pumping expense, the cost \(c_{ij}\) ($) has been predetermined as given in the following table.

Table 3 Transportation costs from plant \(i\) to city \(j\)

<table>
<thead>
<tr>
<th>From Plant (i)</th>
<th>To City (j)</th>
<th>Transportation Cost (c_{ij}) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(1)</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>(4)</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>5</td>
</tr>
<tr>
<td>(2)</td>
<td>(1)</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>(4)</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>10</td>
</tr>
<tr>
<td>(3)</td>
<td>(1)</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>(4)</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>12</td>
</tr>
</tbody>
</table>

The upper limit of the transportation capacity from plant \(i\) to city \(j\) is given as:
Table 4 Transportation capacities from plant \( i \) to city \( j \)

<table>
<thead>
<tr>
<th>From Plant ( i )</th>
<th>To City ( j )</th>
<th>Transportation Capacities ( RU_{ij} ) (million gallons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i )</td>
<td>1</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>150</td>
</tr>
<tr>
<td>( 2 )</td>
<td>1</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>130</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>160</td>
</tr>
<tr>
<td>( 3 )</td>
<td>1</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>160</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>90</td>
</tr>
</tbody>
</table>

For each 1 million gallons of unmet water demand for each city, the penalties are assigned as in the following table:

Table 5 Penalty of the unmet water demand for city 1 to 5

<table>
<thead>
<tr>
<th>City ( j )</th>
<th>Penalty ( P_j ) ($ per million gallons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>180</td>
</tr>
<tr>
<td>3</td>
<td>240</td>
</tr>
<tr>
<td>4</td>
<td>190</td>
</tr>
<tr>
<td>5</td>
<td>230</td>
</tr>
</tbody>
</table>

Suppose there are two sources available in this region, and their maximum supply capacities, production cost and hardness concentration are defined as the follows:

Table 6 Maximum supply capacities, production costs and hardness concentrations of two sources \( a \) and \( b \).

<table>
<thead>
<tr>
<th>Source</th>
<th>Type</th>
<th>Max Supply Capacities (million gallons)</th>
<th>Production Cost ( E_k ) $ (per million gallon)</th>
<th>Hardness Concentration ( H_k ) (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>reservoir</td>
<td>1,700</td>
<td>204</td>
<td>85</td>
</tr>
<tr>
<td>( b )</td>
<td>well</td>
<td>1,400</td>
<td>140</td>
<td>180</td>
</tr>
</tbody>
</table>

And the standard hardness concentration \( H_{stan} \) is set to be 100 ppm.

**SOLUTION HEURISTIC**

The above mentioned model can be rearranged to the following form, i.e., \( b_i \) can be replaced by \( 100 - a_i \), totally the model contains 18 decision variables and 28 constraints, and all the constraints are less than and equal to constraints.

Objective Function (Minimize the total cost + penalty, for one month):

\[
\text{Min} \sum_{i=1}^{3} \sum_{j=1}^{5} c_{ij} x_{ij} + \sum_{i=1}^{3} (E_b + \frac{(E_a - E_b)a_i}{100}) (\sum_{j=1}^{5} x_{ij}) + \sum_{j=1}^{5} P_j (D_j - \sum_{i=1}^{3} x_{ij})
\]
Subject to:

\[
\sum_{j=1}^{5} x_{ij} \leq CaP_i, \text{ for plant } i = 1,2,3 \quad (1)-(3)
\]

\[
\sum_{i=1}^{3} x_{ij} \leq D_j, \text{ for city } j = 1,2,3,4,5 \quad (4)-(8)
\]

\[
x_{ij} \leq RU_{ij}, \text{ for all } i \text{ and } j \quad (9)-(23)
\]

\[
\sum_{i=1}^{3} \left( \frac{1}{100} a_i \right) \left( \sum_{j=1}^{5} x_{ij} \right) \leq Max(R) \quad (24)
\]

\[
\sum_{i=1}^{3} \left( 1 - \frac{1}{100} a_i \right) \left( \sum_{j=1}^{5} x_{ij} \right) \leq Max(P) \quad (25)
\]

\[
(H_a - H_b) \left( \frac{a_i}{100} \right) \leq H_{\tan} - H_b, \text{ for plant } i = 1,2,3 \quad (26)-(28)
\]

\[
x_{ij}, a_i \geq 0
\]

In this study, the commercial software CPLEX® is first applied to the above model. CPLEX is capable of solving linear, nonlinear and integer programming problems by using branch-and-bound algorithm. But when this problem is given to CPLEX, an error message “dual problem is not feasible” appears due to the highly nonlinear function formulation. So applying another algorithm is necessary to solve this model.

**Penalty Methods**

One approach to solving constrained nonlinear programs is to convert them to a series of unconstrained ones. A commonly used scheme for transforming constrained into unconstrained NLPs is penalty methods. It drops constraints of nonlinear program and substitute new terms in the objective function penalizing infeasibility in the form

\[
\min F(x) = f(x) + \mu \sum_i p_i(x)
\]

where \( \mu \) is a positive penalty multiplier and \( p_i \) are penalty functions and in this case, they take the form of \( \max \left\{ 0, g_i(x) - b_i \right\} \). Since all the constraints are \( \leq \)’s. \( g_i(x) \) and \( b_i \) are the left and right hand side of each constraints. Each imposes no penalty when the corresponding constraint is satisfied, but adds a growing cost if it is violated. Also the penalty multipliers can be adjusted according to different scenarios, like either emphasizing water supply or water quality as shown in the following table.

**Table 7 Penalty Multipliers’ Value**

<table>
<thead>
<tr>
<th>Multiplier</th>
<th>Associated Constraint Equations</th>
<th>Value for Water Supply</th>
<th>Value for Water Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1) ~ (3)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>(4) ~ (8)</td>
<td>1~5</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>(9) ~ (23)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>(24), (25)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>(26) ~ (28)</td>
<td>1</td>
<td>1~5</td>
</tr>
</tbody>
</table>
Nelder-Mead Algorithm

Considering the highly ill-conditioned nonlinear model formulation, one of the most popular schemes for unconstrained search without derivatives, the Nelder-Mead algorithm is applied here. Detailed application procedure is described as the following, based on:

Step 0: Initialization. Choose \( n + 1 \) distinct solutions \( x^{(j)} \) as starting set \( \{ y^{(1)}, \ldots, y^{(n+1)} \} \), evaluate \( f(y^{(1)}), \ldots, f(y^{(n+1)}) \), and initialize iteration index \( t \leftarrow 0 \). In this case, \( n = 18 \), and \( y^{(i)} = (a_{1}^{(i)}, a_{2}^{(i)}, a_{3}^{(i)}, x_{11}^{(i)}, \ldots, x_{35}^{(i)}) \); 

Step 1: Centroid. Renumber as necessary to arrange the \( y^{(i)} \) in nonimproving sequence by solution value \( f(y^{(1)}) < f(y^{(2)}) < \ldots < f(y^{(n)}) < f(y^{(n+1)}) \). Then compute best-n centroid

\[
\chi^{(t)} = \frac{1}{n} \sum_{i=1}^{n} y^{(i)}
\]

Step 2: Stopping. If all solution values \( f(y^{(1)}), \ldots, f(y^{(n)}) \) are sufficiently close to centroid objective value \( f(x^{(t)}) \), stop and report the best of \( f(y^{(1)}) \) and \( x^{(t)} \). Here take

\[
\sqrt{\frac{1}{n+1} \sum_{i=1}^{n+1} [f(y^{(i)}) - f(x^{(t)})]^2} < \epsilon = 0.01
\]

Step 3: Direction. Use centroid \( \chi^{(t)} \) to compute away-from-worst move direction

\[
\Delta x^{(t+1)} \leftarrow \chi^{(t)} - y^{(n+1)}
\]

Step 4: Reflection. Try \( \lambda = 1 \) by computing \( f(x^{(t)} + \lambda \Delta x^{(t+1)}) \). If this new value is at least as good as current best \( f(y^{(1)}) \), go to step 5 and expand. If it is no better than second-worst value \( f(y^{(n)}) \), go to step 6 and contract. Otherwise, accept \( \lambda \leftarrow 1 \), and proceed to Step 8.

---

Figure 1 Graphical Representation of Nelder-Mead Movement ---- Reflection
Step 5: Expansion. Try \( \lambda = 2 \) by computing \( f(x^{(t)} + 2\Delta x^{(t+1)}) \). If this value is no worse than \( f(x^{(t)} + 1\Delta x^{(t+1)}) \) fix \( \lambda \leftarrow 2 \), and otherwise set \( \lambda \leftarrow 1 \). Then proceed to step 8.

![Figure 2 Graphical Representation of Nelder-Mead Movement ---- Expansion](image)

Step 6: Contraction. If reflection value \( f(x^{(t)} + 1\Delta x^{(t+1)}) \) is better than worst current \( f(y^{(n+1)}) \), try \( \lambda = \frac{1}{2} \) by computing \( f(x^{(t)} + \frac{1}{2}\Delta x^{(t+1)}) \). If not, try \( \lambda = -\frac{1}{2} \) by evaluating \( f(x^{(t)} - \frac{1}{2}\Delta x^{(t+1)}) \). Either way, if the result improves on worst current \( f(y^{(n+1)}) \), fix \( \lambda \) at the \( \pm \frac{1}{2} \) tried and proceed to Step 8. Otherwise, go to Step 7 to shrink.

![Figure 3 Graphical Representation of Nelder-Mead Movement ---- Contraction](image)

Step 7: Shrinking. Shrink the current solution set toward best \( f(y^{(1)}) \) by

\[
y^{(i)} = \frac{1}{2} (y^{(1)} + y^{(i)}) \quad \text{for all } i = 2, \ldots, n + 1
\]

Then compute new \( f(y^{(2)}) \), \ldots, \( f(y^{(n+1)}) \), advance \( t \leftarrow t + 1 \), and return to Step 1.
Step 8: Replacement. Replace worst \( y^{(n+1)} \) in the solution by

\[
x^{(t)} + \lambda \Delta x^{(t+1)}
\]

Then advance \( t \leftarrow t + 1 \) and return Step 1.

Note that we require all the decision variables be greater or equal to zero. So during the calculation, if any variable is reflected to be less than zero, we should attach a very large positive number to function value \( f(y^{(i)}) \) to discourage this movement and keep the solution feasible. The computation result shows that any negative trespassing will be followed automatically by contraction moves which will eventually keep it inside the solution domain.

**NUMERICAL RESULTS AND DISCUSSIONS**

The initial distinct 19 solutions are simply provided by the upper bound for each arc as the primary transportation values. The above mentioned algorithm was coded in Visual C++ 6.0 and the resulting iteration times vs. function value is shown in Figure 5 and Figure 6. The computational time is about 8-9 seconds for each case.

For the water supply scenario, Figure 5 shows when different values of the demand penalty multiplier’s are used, all the corresponding objective function values (with penalty) converge to constant values as iterations continues. And it is also found that as the penalty multiplier’s value increases, the deviation of the water demand which is defined as

\[
Dev = \sqrt{\sum_{j=1}^{5} \left( \sum_{i=1}^{3} x_{ij} - D_j \right)^2}
\]

continues decreasing as \( \mu_2 \) reaches 4, and then increasing when \( \mu_2 \geq 5 \). Figure 7 shows Final objective function value (without penalty) vs penalty multipliers \( \mu_2 \) for water supply scenario. When \( \mu_2 \) is ranging from 3 to 5, the objective function value doesn’t vary much, so \( \mu_2 = 4 \) can be considered a reasonable good choice.
For the water quality scenario, which is harder-to-realize objective, Figure 6 shows the function values converges when the iteration step proceeds as in the previous case. The deviation still decreases as we increase the multiplier’s value, when $\mu_5$ equal to 4, the deviation reaches its minimal values. Again Figure 8 shows $\mu_5 = 4$ can be considered a reasonable multiplier’s value for the same reason mentioned above. This shows the Nelder-Mead algorithm works well for this model and the solutions go to desired direction.

CONCLUSIONS

From the research present above, the following conclusions can be drawn:

- The basic transportation model is extended to nonlinear form considering various financial/social, physical and environmental factors;
- The application of penalty methods and penalty multipliers’ adjustment is suitable for different scenarios;
- Improved Nelder-Mead derivative free search algorithm generates near optimal solution for the problem in reasonable computational time.

The future work will be aimed at applying the proposed model to real world problem. In regional water supply system, the extended nonlinear transportation model will provide a simpler, more efficient tool to assess the operational polices and system reliability. The computational efficiency will be improved as well by using Nelder-Mead Algorithm.
Figure 5 Iteration vs. Function Value for water supply scenario

Table 8 Deviations of water demand for each city

<table>
<thead>
<tr>
<th></th>
<th>$\mu_2 = 1$</th>
<th>$\mu_2 = 2$</th>
<th>$\mu_2 = 3$</th>
<th>$\mu_2 = 4$</th>
<th>$\mu_2 = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1 = 180$</td>
<td>200</td>
<td>178</td>
<td>166</td>
<td>183</td>
<td>186</td>
</tr>
<tr>
<td>$D_2 = 100$</td>
<td>120</td>
<td>123</td>
<td>113</td>
<td>104</td>
<td>102</td>
</tr>
<tr>
<td>$D_3 = 140$</td>
<td>179</td>
<td>147</td>
<td>151</td>
<td>148</td>
<td>149</td>
</tr>
<tr>
<td>$D_4 = 120$</td>
<td>133</td>
<td>139</td>
<td>122</td>
<td>126</td>
<td>125</td>
</tr>
<tr>
<td>$D_5 = 260$</td>
<td>301</td>
<td>284</td>
<td>269</td>
<td>268</td>
<td>268</td>
</tr>
<tr>
<td>Deviation</td>
<td>28.88</td>
<td>17.42</td>
<td>10.68</td>
<td>6.15</td>
<td>6.48</td>
</tr>
</tbody>
</table>
Figure 6 Iteration vs. Function Value for water quality scenario

Table 9 Deviations of fraction number of reservoir water usage

<table>
<thead>
<tr>
<th></th>
<th>$\mu_5 = 1$</th>
<th>$\mu_5 = 2$</th>
<th>$\mu_5 = 3$</th>
<th>$\mu_5 = 4$</th>
<th>$\mu_5 = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$ = 84</td>
<td>0</td>
<td>0</td>
<td>33</td>
<td>59</td>
<td>51</td>
</tr>
<tr>
<td>$a_2$ = 84</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td>64</td>
<td>57</td>
</tr>
<tr>
<td>$a_3$ = 84</td>
<td>0</td>
<td>77</td>
<td>73</td>
<td>68</td>
<td>65</td>
</tr>
<tr>
<td>Deviation</td>
<td>84</td>
<td>68.70</td>
<td>52.75</td>
<td>20.66</td>
<td>26.95</td>
</tr>
</tbody>
</table>
Figure 7 Final Objective Function Value vs. Penalty Multipliers $\mu_2$ for water supply scenario

Figure 8 Final Objective Function Value vs. Penalty Multipliers $\mu_5$ for water quality scenario
REFERENCES


